Hydrology Technical Note 2

CHAPTER 1

THE MODIFIED ATT-KIN ROUTING MODEL

INTRODUCTION

This Tech Note presents the math model to be used for channel flood routing. The model, a Modified Att-Kin (attenuation-kinematic) procedure, replaces the convex procedure which is in the previous Technical Release 20. The TR-20 computer program was developed in 1964 and incorporated manual flood routing procedures, but current technology and changing requirements for the use of TR-20 called for improved yet uncomplicated procedures. The Modified Att-Kin procedure fills this need.

The Modified Att-Kin procedure for valley flood routing is based on the Att-Kin procedure described in TR-66 (1981). The TR-66 Att-Kin procedure was modified to conform to the structure of the existing TR-20 computer is an interim procedure to be used until more complete routing models are included.

The Modified Att-Kin procedure does not alter the basic form of the original convex routing equation. It provides a better physical basis for determining the routing coefficient while using a numerical technique that satisfies the intended math model. The physical basis of the Modified Att-Kin procedure is twofold: (1) it satisfies the time to peak of the hydrograph throughout the valley; and (2) it satisfies the conservation of mass at the time to peak. To accomplish these items, the Modified Att-Kin method combines the best features of both the storage and the kinematic math models.

DESCRIPTION OF MATH MODELS

Before describing the Att-Kin procedure a brief description of the physical behavior of natural flood waves is necessary.

Linsley, Kohler and Paulhus (1982) describe two general classes of natural flood waves. The first includes waves in which forces of momentum and acceleration control. These waves are fast rising, and occur frequently in streams of steep gradient. The second class includes flood waves in which friction is the predominant force. Slow rising flood waves on streams of mild gradients are examples of this class.

The movement of a flood wave down a stream system may be described in terms of translation and reservoir effects. These represent characteristics of the two general classes of natural flood waves. Translation involves maintaining the same hydrograph shape as the flood wave moves downstream. The reservoir effect involves use of valley storage to reduce the peak flow and change the shape of the hydrograph.

The Att-Kin procedure is based on storage and kinematic models to reflect the reservoir and translation effects on natural flood waves, respectively.

The storage math model is physically related to reservoir routing applications. The mathematical behavior produces instantaneous translation with maximum attenuation for the storage involved. Closed form solutions can be developed for analytic inflow hydrographs with linear storage-discharge relationships. Simple numeric techniques can be constructed and easily solved that satisfy the math model without any stability problems. However, care must be taken not to needlessly subdivide the reach, because the numeric results then approach a kinematic rather than storage math model solution.

The kinematic math model is physically related to wave propagation situations. The mathematical behavior produces translation for any discharge with absolutely no attenuation. Simple closed-form solutions can be done for any type of inflow hydrograph and nonlinear storage-discharge relationship. Numeric models can be constructed and solved that satisfy the math model.

Proper unsteady flow models describing the passage of flood hydrographs through a reach must at least be a combination of both kinematic for translation and storage for attenuation. The full dynamic equations simultaneously account for both effects, but they are very difficult to apply to general field situations. However, hydrologic routing (coefficient) procedures which are simpler than unsteady flow models, can be used for most field application if (1) a math model is chosen that describes the important physical processes, (2) the routing coefficients are determined for each application to reflect the actual site-specific physical relationships, and (3) a numeric technique is constructed that satisfies the chosen math model.

<u>Att-Kin Procedure</u>

A thorough discussion of the Att-Kin procedure is presented in TR-66. This brief discussion is included as an introduction to the procedure.

For each channel (valley) reach, the Att-Kin procedure routes an inflow hydrograph through the reach using a storage math model. The procedure then positions the peak in time and distorts the storage routed hydrograph using kinematic model. These routings cannot be done simultaneously but must be done as a linear combination of the storage and kinematic models. The combined routing is done so the outflow hydrograph satisfies the conservation of mass equation at the time to peak of the outflow hydrograph. Figure 1-1 illustrates the two models which make up the Att-Kin procedure. The storage routing provides attenuation but does not describe translation; the kinematic routing provides translation and distortion but does not attenuate the peak.

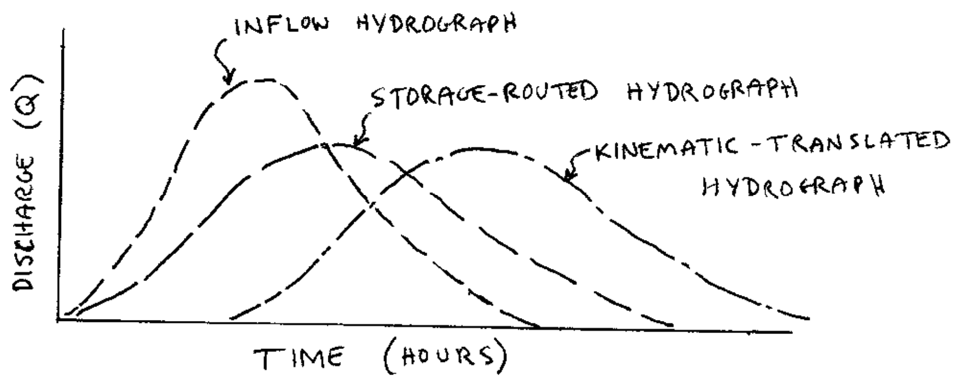


Figure 1-1. Hydrographs resulting from storage and kinematic translation.

The inflow hydrograph is storage routed using the storage indication method to solve the integral form of the conservation of mass equation using a single-valued valley storage-discharge relationship. The equations which reflect attenuation are:

$$(QI - QO)(t2 - t1) = S(t2) - S(t1) \qquad \text{Eq. 1-1}$$

$$Q(t) = k\,[\,S(t)\,]^{m} \qquad \text{Eq. 1-2}$$

where: t1 = time at the beginning of the interval
t2 = time at the end of the interval
QI = the average inflow over the time interval t2-t1
QO = the average outflow over the time interval t2-t1
S(t) = valley storage in the reach at time t
Q(t) = outflow at the downstream end of the reach at time t
k and m = coefficient and exponent for the single valued storage - discharge relationship

The storage routed hydrograph is then translated and distorted by kinematic routing. Kinematic routing solves the differential form of the conservation of mass equation using a single-valued flow area-discharge relationship. The equations used to effect translation are:

$$\partial Q / \partial X + \partial A / \partial t = 0 \qquad \text{Eq. 1-3}$$

where:
- Q = discharge, cfs
- X = distance, ft
- A = area, ft
- t = time, sec

and: $$Q = xA^{m} \qquad \text{Eq.1-4}$$

where:
- Q = discharge at any distance and time
- A = Area at any distance and time
- x and m = coefficient and exponent for the single valued flow area-discharge relationship

These two routing equations (Equations 1-1 and 1-3) require two separate single-valued discharge relationships. The two single-valued functions can be related by

$$S = LA \qquad \text{Eq. 1-5}$$

where:
- S = valley storage
- L = valley length
- A = valley cross-section area

An inherent assumption made in Equation 1-5 is that a constant or representative cross section shape for the length of the reach exists. The flow area-discharge relationship can be determined from the valley storage-discharge relationship by

$$Q = k\,S^{m} = k\,L^{m}\,A^{m} = xA^{m} \qquad \text{Eq. 1-6}$$

Some, but not all, of the valley storage results in attenuation of the peak. Part of the storage is used for translation; that is, when the attenuated hydrograph is kinematic routed, some of the valley storage is filled. Therefore, the storage volume used in the storage-routing portion of the Att-Kin procedure is related to the valley storage-discharge relationship by a constant of proportionality, as follows:

$$Qpo = k \; [\, C2 \;\; Spo \,]^{m} \qquad \text{Eq. 1-7}$$

$$C2 = Vs \; / \; Spo \qquad \text{Eq. 1-8}$$

Where:
- C2 = constant of proportionality
- Spo = maximum valley storage in the reach during passage of and assumed coincident with the outflow peak of the kinematic translated hydrograph.
- Qpo = peak of the outflow hydrograph
- Vs = net storage used in computing the outflow peak by the storage routing math model.

Vs can be determined from the relationship:

$$Vs = Vo - (\, Vt + Vd \,) \qquad \text{Eq. 1-9}$$

Each term in equation 1-9 is described below and illustrated on Figure 1-2.

Vo = the maximum valley storage; difference between the accumulated inflow volume and the accumulated outflow volume at the time to peak of the outflow hydrograph.

$$Vo = \int_{o}^{tpo} Q(t,o)\; dt \; - \int_{o}^{tpo} Q(t,L)\; dt \qquad \text{Eq.1-10}$$

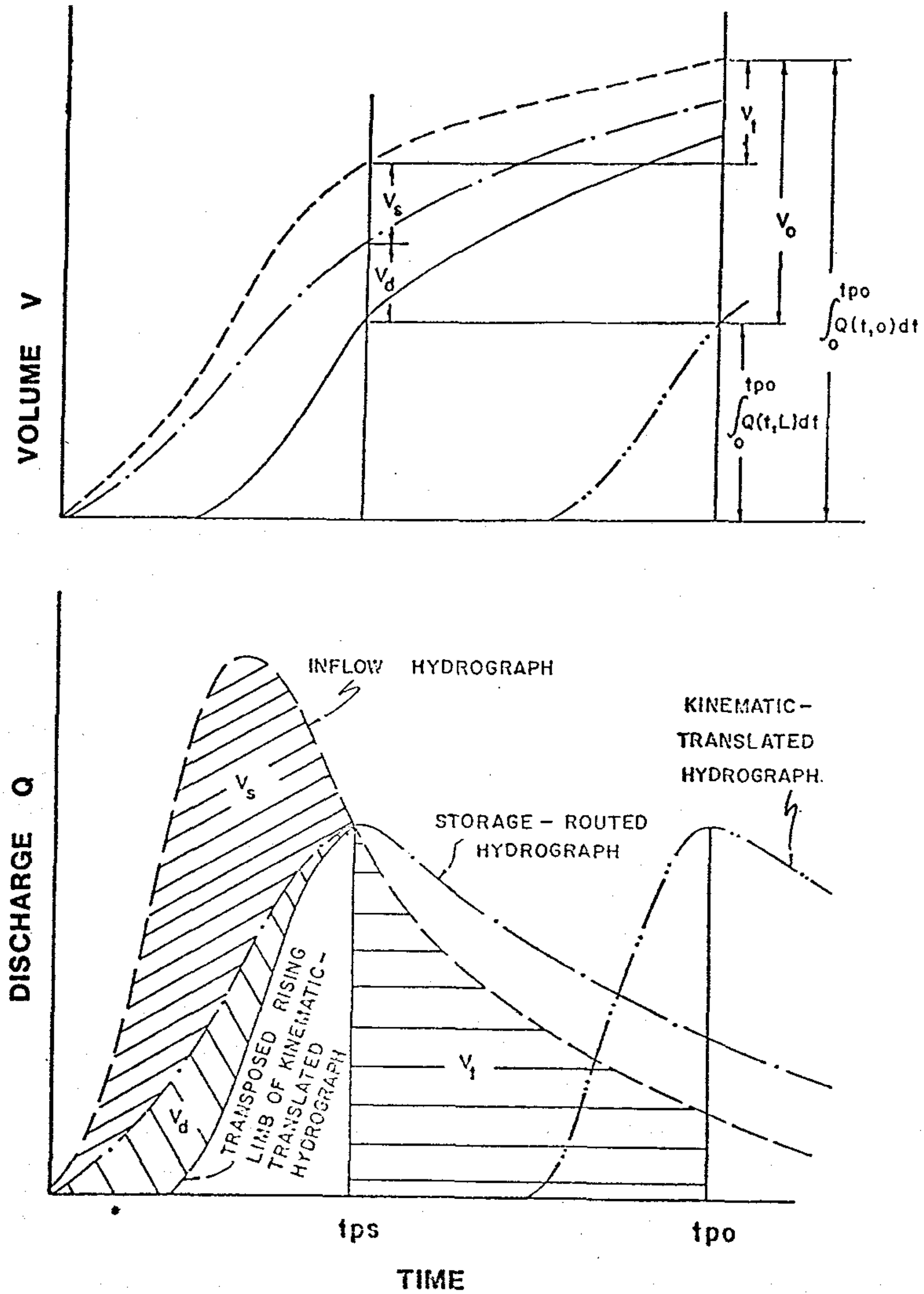


Figure 1-2. Volumes used in the Att-Kin routing procedure.

where: $Q(t,o)$ = discharge at time t at upstream end of reach
$Q(t,L)$ = discharge at time t at downstream end of reach
tpo = time to peak discharge of the kinematic translated hydrograph.

The volume Vt is the additional inflow that occurs between the times of the storage and kinematic-routed peaks and is defined as

$$Vt = \int_{tps}^{tpo} Q(t,o)\, dt \qquad \text{Eq.1-11}$$

where tps = time to peak of the storage routed hydrograph.

The distortion volume is defined as

$$Vd = \int_{o}^{Qo} [(L / C1) - (L / Co)]\, dQ \qquad \text{Eq. 1-12}$$

where:
L = reach length, ft
Q = peak outflow discharge, cfs
C1 = m(Q/A), ft/sec.
Co = m (Qo /Ao) ft/sec
m = exponent of flow area-discharge relationship.
Ao = flow area associated with Qo

The distortion volume (Vd) is a result of steepening the rising limb of the storage routed hydrograph. Offsetting the entire hydrograph by the kinematic travel time of the peak would not be correct. Each discharge travels at a different celerity resulting in a distortion of the rising limb of the hydrograph. This distortion is the difference between the travel times of the instantaneous discharges (L/C1) and peak discharge (L/Co). Integrating these differences over the rising limb of the hydrograph gives Vd. (This is illustrated by placing the kinematic-routed hydrograph so that its peak is coincident with the storage-routed hydrograph). The volume Vt is the additional inflow that occurs between the times of the storage and kinematic-routed peaks. The storage volume (Vs) is that part of the valley storage which causes attenuation.

The solution of the Att-Kin procedure is iterative and involves selecting a value of C2 (defined by equation 1-8). The inflow hydrograph is then storage-routed.

Vd is computed from equation 1-12 and the hydrograph is translated through the kinematic routing. Vs is computed from equation 1-9 and compared with Vs resulting from the storage routing. If the two values are not equal then a new value of C2 is assumed and the process repeated until they are equal. At this point Vo equals Spo. This iterative procedure insures a balance of the conservation of mass at the time of the peak outflow.

Modified Att-Kin Procedure

The Modified Att-Kin procedure replaces the Convex procedure but uses a similar derivation with three exceptions. (1) An arithmetic average of flow at consecutive time intervals is assumed for the outflow, (2) the determination of the routing coefficient, Cr , is related to the site-specific physical processes, and (3) the numeric model is constructed to satisfy the math model.

The structure of the TR-20 computer program dictates a routing equation of the form

$$O2 = Cr\ I1 + (1 - Cr)\ O1 \qquad \text{Eq. 1-13}$$

where: $O1$ = the outflow at time 1, cfs
$O2$ = the outflow at time 2, cfs
$I1$ = the inflow at time 1, cfs
Cr = coefficient for routing

The equation for conservation of mass can be written as

$$I - O = (S2 - S1) / \Delta t \qquad \text{Eq. 1-14}$$

and solved by assuming I1 is the average inflow over the time interval t and (O2 + O1)/2 is the average outflow over the same time interval. Making this assumption results in a simpler routing equation involving I1 , O1 and O2 rather than I1 , I2 , O1, and O2. If Δ t is chosen such that there is not a large difference between consecutive discharges this is a reasonable assumption. This results in:

$$I1 - (O2 + O1) / 2 = (S2 - S1) / \Delta t \qquad \text{Eq. 1-15}$$

Let S = KO, where K approximates the slope of storage-outflow curve. The best straight line estimate of the slope is the tangent to the curve at the peak inflow as this represents the wave celerity or kinematic wave velocity shown on Figure 1-3.

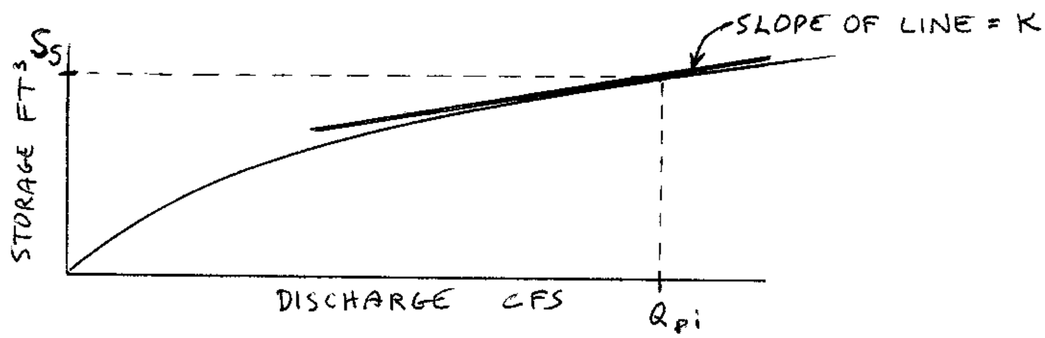


Figure 1-3. Storage-outflow slope used in the Modified Att-Kin routing

Equation 1-15 becomes

$$I1 - (O2 + O1) / 2 = K (O2 - O1) / \Delta t \qquad \text{Eq. 1-16}$$

Expressing Eq. 1-16 in the form of Eq.1-13 (see Theory Development for the Modified Att-Kin Routing equations 1-20 thru 1-22) yields an equation for Cr.

$$Cr = 2 \Delta t / (2 K + \Delta t) \qquad \text{Eq. 1-17}$$

K is computed from the storage and discharge information (Figure 1-3) using the equation

$$K = Ss / Qpi / m$$ (see fig. 1-3)

$$\text{or } K = L / (m V) \qquad \text{Eq. 1-18}$$

where L = reach length, ft
V = velocity of discharge equal to Qpi, ft/sec
m = rating curve exponent

Since the actual outflow peak and associated storage is not known it must be estimated before the routing. The equations and their derivation are shown in Theory Development equations 1-23 to 1-28.

The value of K computed in Equation 1-18 is used in equation 1-17 to solve for Cr. Cr is not allowed to exceed 1. The routing is done using equation 1-13. The time difference between the inflow peak and the outflow peak is Δtps. The value of tps is a multiple of the main time increment used in the reach routing. The resulting hydrograph is then positioned in time so the difference between the time to peak of inflow and outflow is

$$\Delta tp = (Spi - Spo) / (Qpi - Qpo) / 3600 \qquad \text{Eq. 1-19}$$

where: Spi = valley storage associated with the inflow peak, ft
Spo = maximum valley storage in the reach during the passage of and assumed coincident with the outflow peak,
Qpi = peak of the inflow hydrograph including baseflow,
Qpo = peak of the outflow hydrograph including baseflow,
Δtp = elapsed time between the inflow and outflow peaks, hrs

The equation for Δtp is developed from the kinematic travel time assuming that peak discharge varies linearly with time during its movement through the reach.

The time to peak of the kinematic routed hydrograph (Δtp) is used to position the outflow hydrograph if $\Delta tp > \Delta tps$.

THEORY DEVELOPMENT FOR THE MODIFIED ATT-KIN ROUTING

The development of the Modified Att-Kin routing method involves the definition of Cr , the routing coefficient used in the routing equation 1-13.

The equation for the storage routing is repeated for the following derivation of the routing equation.

$$I1 - (O2 + O1) / 2 = K (O2 - O1) / \Delta t \qquad \text{Eq. 1-16}$$

where:
- I1 = average inflow discharge, cfs
- O2 = outflow discharge at time 2, cfs
- O2 = outflow discharge at time 1,cfs
- K = approximate slope of the storage discharge curve at the peak outflow.

Multiply equation 1-16 by 2 Δt.

$$2 \Delta t \; I1 - \Delta t (O2 + O1) = 2 K (O2 - O1) \qquad \text{Eq. 1-20}$$

Add 2KO and Δt O2 to both sides of equation 1-20.

$$2 \Delta t \; I1 - \Delta t \; O1 + 2 K \; O1 = 2 K \; O2 + \Delta t \; O2$$

or

$$2 \Delta t \; I1 + (2 K - \Delta t) O1 = (2 K + \Delta t) O2 \qquad \text{Eq.1-21}$$

Divide both sides of equation 1-21 by 2K + Δt and solve for O2.

$$O2 = \left(\frac{2 \Delta t}{2 K + \Delta t}\right) I1 + \left(\frac{2 K - \Delta t}{2 K + \Delta t}\right) O1$$

Substitute $\Delta t - 2\Delta t$ for $-\Delta t$,

$$O2 = \left(\frac{2\,\Delta t}{2\,K + \Delta t}\right) I1 + \left(\frac{2\,K + \Delta t - 2\Delta t}{2\,K + \Delta t}\right) O1$$

$$O2 = \left(\frac{2\,\Delta t}{2\,K + \Delta t}\right) I1 + \left(1 - \frac{2\,\Delta t}{2\,K + \Delta t}\right) O1 \qquad \text{Eq. 1-22}$$

Substituting Cr for $2\Delta t\ /\ (2\,K + \Delta t)$, equation G-22 becomes

$$O2 = Cr\ I1 + (1 - Cr)\ O1$$

Before Cr can be computed, K is needed. The equations which are used in the TR-20 program to compute K are derived below.

The single valued storage - discharge relationship is calculated from the flow area-discharge relationship and the reach length appearing in equation 1-6.

The next step is to compute the length factor, k*. The equation defining k* is:

$$k^* = Qpi\ /\ k\ /\ VI^m \qquad \text{Eq. 1-23}$$

where: Qpi = peak of runoff hydrograph, cfs
VI = volume of runoff hydrograph, ft
k, m = coefficients of storage-discharge relationship.

The value of k* appears in output Summary Table 2 under heading, LENGTH FACTOR (K*).

The denominator of equation 1-23 represents the discharge at which all the inflow volume is stored in the reach. This discharge is inversely related to the reach length. For short reach lengths, this discharge could be extremely large and for long reach lengths, this discharge could be comparable to the peak of the runoff hydrograph, Qi. Therefore, k* is directly related to reach length.

For example, as the reach length increases, k* increases.

An estimate of the peak outflow is needed in order to compute K, the slope of the storage-discharge curve. This estimated peak, denoted Qref, is a reference discharge and is equal to

Qref = Qpi   if k^* is less than or equal to 1.0

or Qref = Qpi / k^* if k^* is greater than 1.0.

The value of K is computed at the reference discharge, Qref.

$$K = L / m / Vref \qquad \text{Eq. 1-24}$$

where L = reach length, ft
Vref = velocity of reference discharge, ft/sec
m = rating curve exponent

The velocity of the reference discharge is computed from the rating curve equation $Q = x A^m$ and is

$$Vref = x^{1/m} \; Qref^{(1 - 1/m)}$$

Using this value of K, Cr is computed from equation 1-17.

$$Cr = (2 \Delta t \; (3600)) / (2 K + \Delta t \; (3600))$$

where Δt is the main time increment in hours. This value of Cr is shown in Summary Table 2 as ATT-KIN COEFF (C). A discussion of the relationship of Cr , Δt, K and L can be found in Chapter 2.

The inflow hydrograph is then routed according to equation 1-13.

The difference in peak times for the inflow and outflow hydrographs is then computed. This time represents the travel time of the peak discharge through the reach based on the storage routing. The kinematic routing is then accomplished by computing the kinematic travel time, Δtp. Equation 1-19 is derived to a different form in the TR-20 program.

Substituting

$$Spi = \left(Qpi / k\right)^{1/m}$$

$$Spo = \left(Qpo / k\right)^{1/m}$$

into Equation 1-19 yields Equation 1-25.

$$\Delta tp = \frac{Spo \left[\left(Qpi / Qpo \right)^{1/m} - 1 \right]}{Qpo \left[\left(Qpi / Qpo \right) - 1 \right]} \qquad \text{Eq. 1-25}$$

where: k,m = coefficient and exponent of valley storage-discharge relationship.

Other variables are as defined with equation 1-19.

If Δtp is greater than Δtps then the outflow is positioned such that the peak of the outflow hydrograph occurs Δtp hours after the peak of the inflow hydrograph. If Δtp is less than Δtps then the positioning of the outflow hydrograph remains unchanged. In TR-20, the positioning of the outflow hydrograph is done using a value of Δtp rounded to the nearest multiple of the main time increment.

If x and m coefficients are entered as input data and the m value is within the program limits (1.0 to 2.0) see Chapter 2, the routing is completed for the reach. If m is outside the program limits, an iteration technique is used to route the hydrograph through the reach such that the area corresponding to the outflow peak discharge falls within 5% of the area as computed from the discharge-flow area equation (defined by the input x and m). In these iterations if the input m is greater than 2.0 then the m used in the routing is 2.0 and if the input m is less than 1.0 then m used in the reach routing is 1.0. The iteration technique makes a correction to x at the end of each routing trial to put the next trial closer to the final result. Figure G-4 illustrates how the program handles the routing with an input m less than 1.0. The final routing has m = 1.0 and the outflow falling on (or within the program tolerance of) the discharge-area curve as defined by the input x and m. This insures that the peak outflow is the peak corresponding to the maximum valley storage or $Qpo = k \, (Spo)^{m}$ where variables are defined in Equation 1-7.

If the number of iterations equals 10, the user is notified in Summary Table 2. If a rating table is entered as cross section data, a similar iteration technique is used with the added requirement that the m used in the final routing trial must be within 0.05 of the m at the peak outflow discharge. At the end of each routing trial, corrections to the values of m and x are made to put the next routing trial closer to the final result. A maximum of 10 iterations is set in the program.

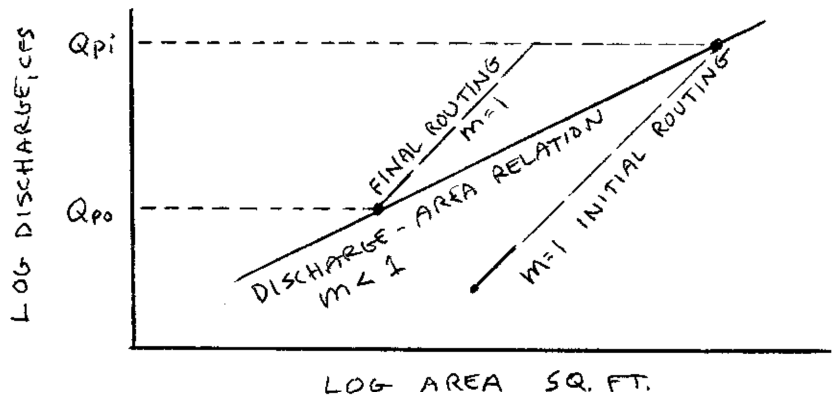


Figure 1-4. Illustration of routing iterations in TR-20 program.

USE OF THE MODIFIED ATT-KIN METHOD WITH MEANDERING CHANNELS

The effect of a meandering channel on flood routing can be significant depending on the size of the channel and degree of meandering. When computing storage in the reach, the cross section flow area (from the rating table) and reach length are routinely multiplied. In a meandering channel, the channel length can be significantly longer than the length of the flood plain which will result in more volume stored in the reach and more peak discharge attenuation. A procedure has been developed to account for this additional storage if the user feels it is significant.

Information needed in order to use this procedure includes a measurement of channel length and flood plain length for the reach and a rating table consisting of several elevations with associated total cross section discharge and total cross section flow area. A cross section plot (distance versus elevation) is often useful.

The procedure is based on use of the channel length multiplied by the channel flow area to get channel storage and use of the flood plain length and flood plain flow area to get flood plain storage. These storages are then added to get total reach storage. An elevation is selected based on cross section information which will be the dividing line between channel and flood plain area. In an ideal cross section where both channel banks are at the same elevation and the flood plain elevations are all at or above the bank elevation, the channel bank elevation is the obvious selection. In actual cross sections the elevation to select is not so obvious. Examination of a cross section plot is often useful. An elevation termed "low ground" which is defined as the low point of the cross section outside or including the channel banks is usually the elevation where the flood plain area begins to increase and overshadow the channel area. This elevation is recommended unless the cross section plot indicates there is a better dividing elevation.

This procedure is best illustrated through an example.

A reach has a channel length of 6000 feet and a flood plain length of 5000 feet. The meander factor is 6000 / 5000 or 1.2. The "low ground" elevation at the cross section is 403. The rating table is as follows.

Elevation	Discharge (Q) cfs	Flow Area (A) sq.ft.
400	0	0
401	10	3
402	25	8
403	50	15
404	100	30
405	200	75
406	400	125
407	600	175

The following table shows computation results. The object of the table is to develop a net flow area which when multiplied by the channel length will give the correct reach storage at a particular elevation in the cross section.

Elevation	Q	A	Channel Area	FL PL Area	Revised FL PL Area	Net Area
400	0	0	0	0	0	0
401	10	3	3	0	0	3
402	25	8	8	0	0	8
403	50	15	15	0	0	15
404	100	30	15	15	12.5	27.5
405	200	75	15	60	50	65
406	400	125	15	110	91.7	106.7
407	600	175	15	160	133.3	148.3

The flood plain area (FL PL Area) is the difference in total area (A) and channel area. The column for Revised FL PL Area is the FL PL Area divided by the meander factor (1.2). The Net area column is the sum of the channel area and the Revised FL PL Area columns.

When completing the flood routing for this reach using the Modified Att-Kin method, the elevation, discharge, and net area columns of this table should be used along with the channel length to represent the reach length.

References

1. USDA, Soil Conservation Service, Technical Release 66, Simplified Dam-Breach Routing Procedure, September, 1985.

2. Linsley, Kohler, Paulhus. 1982. Hydrology for Engineers, Third Edition, p. 264-268, McGraw Hill Book Co., New York.

3. Comer, Theurer, and Richardson, 1981. The Modified Attenuation-Kinematic (ATT-KIN) Routing Model, Proceedings of the International Symposium on Rainfall-Runoff Modeling, Mississippi State University, Water Resources Publications, Copyright 1982.

CHAPTER 2

RATING CURVE COEFFICIENTS x AND m and REACH LENGTH GUIDELINES

DEFINITION AND SIGNIFICANCE OF m

The exponent m has a physical meaning which can add to the understanding of the Modified Att-Kin routing procedure. The discharge-flow area relationship for simple cross sections (rectangular, triangular, trapezoidal) may be fit by a power curve function of the form

$$Q = x A^{m} \qquad \text{Eq. 2-1}$$

where variables are as defined in Eq. 1-4.

According to hydraulic theory, the speed at which a flood wave travels downstream is called the celerity and is equal to the slope of the discharge-flow area curve at a given discharge. In equation form

$$C = dQ / dA \qquad \text{Eq. 2-2}$$

where C is celerity, ft/sec.

Differentiating equation 2-1 with respect to area results in

$$C = dQ / dA = x\, m\, A^{m-1} = m\,(x A^{m}) / A = m\, Q / A \qquad \text{Eq. 2-3}$$

$$C = m\, V$$

where: V = average velocity, ft/sec.

The exponent m is therefore a factor relating average velocity and wave velocity (celerity). The wave velocity is important to the Modified Att-Kin routing method in computation of the Modified Att-Kin routing coefficient and the travel time of the peak discharge through the reach. For details on routing equations see Chapter 1.

The value of m used in the routing has a major effect on the calculation of travel time of the hydrograph through a reach, K. The larger the value of m, the shorter the travel time. Therefore the selection of m can affect the timing of peaks in any reach and can affect the way in which tributary hydrographs add to the peak of the hydrograph on the main stream.

The TR-20 program has internally set limits on the value of m used in reach routings. If the m is too large or too small there is a weakness in both the math and numeric models. A value of m less than 1.0 is unrealistic because the wave velocity (celerity) is less than the average flow velocity. Therefore, the m value used in any given routing is not allowed to be less than 1.0.

To set an upper limit for the value of m, test routings were made with a range of m values from 1.0 to 2.8. The attenuation and peak travel times for each routing were compared with those using an m of 5/3.

This value of m was assumed to provide the most accurate routing. m of 5/3 may be obtained from the following derivation.

The familiar Manning's equation for discharge be rearranged to the following :

$$Q = \frac{1.49\ s^{1/2}}{n\ p^{2/3}}\ A^{5/3} \qquad \text{Eq. 2-5}$$

where:
- n = Manning's roughness coefficient,
- s = hydraulic gradient
- p = wetted perimeter, ft,
- A = flow area, ft , and Q = discharge in cfs.

For cross sections where the hydraulic radius (A/p) can be approximated by the depth (wide channel with bottom width = B), the equation takes the form:

$$Q = x\ A^{m}$$

where: $x = (\ 1.49\ s^{1/2}\ )\ /\ (\ n\ B^{2/3}\ )$ and $m = 5/3$

From the test routings, if m was greater than 2.0, the travel time of the peak flow and the amount of attenuation reduced significantly when compared to the routing where m was 5/3. Therefore the upper limit for m is set at 2.0.

The discussion above concerning celerity and Manning's equation dealt with simple cross section shapes but is also applicable to most cross sections with one segment such as a natural channel. Discharge-area plots for these types of cross sections generally exhibit changes in slope and attempting to fit one power curve to the data can result in significant differences between the two curves..

If the slope of the discharge-area curve (plotted on log-log paper) is greater than one then the average velocity at the cross section is increasing with increasing discharge. If the slope should happen to be less than one, then average velocity is decreasing with increasing discharge. The third possibility is that the slope equals one and the velocity is constant regardless of discharge. For any single segment of a multi-segment cross section, the average velocity should increase with increasing discharge. However, when adding the discharge and area for all segments to get the total discharge and total area at a cross section there can be elevations where the average velocity actually decreases as flow increases. The most common occurrence of this is at discharges exceeding channel capacity with shallow depth in the floodplain for which the velocity in the channel is much larger than the velocity in the floodplain.

The slope of the discharge-area curve where the average velocity decreases (which can be significantly less than one) does not represent the true celerity or wave propagation speed. For this reason, a procedure to calculate m at a cross section was developed. The procedure is described next.

CALCULATION OF m FROM RATING TABLE DATA

The TR-20 user has the input option of entering a rating table to represent flow in a reach. Data required include elevation, discharge, and area for a minimum of three discharges (two greater than zero) to a maximum of twenty discharges (nineteen greater than zero). Elevation, discharge and area must all increase from one value to the next in the table. If any value of flow or area is the same or decreasing there will be an error message.

When a rating is entered, m is computed at each discharge of the table. This procedure will reflect changes in slope of the Q and A curve typically occurring above the bankfull discharge and at large flood plain discharges.

The equations used to compute m are

$$m = S(2,3) \qquad \text{for} \quad Q < Q(3)$$

$$m(I) = \frac{Q(3)\ S(2,3) + \ldots\ (Q(I)-Q(I-1))\ S(I-1,I)}{Q(I)}$$

$$\text{for } Q(3) < Q < Q(I) \qquad \text{Eq. 2-6}$$

where: $m(I)$ = m at flow number I
Q = discharge, cfs
$Q(\ )$ = discharge at subscript point number, $Q(1) = 0$
$S(2,3)$ = log-log slope of discharge-area curve between points 2 and 3
$S(I-1, I)$ = log-log slope of discharge-area curve between points I-1 and I.

m calculated this way can be considered a weighted slope.

The exponent m is extrapolated (log-log) from the last two tabulated points for discharges larger than the last tabulated value.

When m is desired at a nontabulated point, m is linearly interpolated on a log-log basis.

If the log-log slope between each two consecutive points in a rating table is the same then m will be the same for all discharges, in other words, one power will fit the data accurately. When the log-log slope of the discharge-area curve changes then m will change. How m is used in the reach routing is described in Chapter 1.

An example of calculation of m for a rating curve follows.

Discharge, cfs	Area sq. ft.
0	0
200	110
500	200
870	800
1500	1200

Channel capacity is 500 cfs. At discharges higher than 500 cfs, the average velocity reduces due to low flood plain velocity. At discharges above 870 cfs, the average velocity increases again with deeper flow in the flood plain. The slope (log-log) between each pair of consecutive points is calculated:

$$S(2,3) = \log (500/200) / \log (200/110) = 1.5$$

$$S(3,4) = \log (870/500) / \log (800/200) = 0.4$$

$$S(4,5) = \log (1500/870) / \log (1200/800) = 1.3$$

For all discharges at or below 500 cfs, $m = 1.5$.

At a discharge of 870 cfs,

$$m = \frac{(500 \times 1.5) + ((870 - 500) \times 0.4)}{870} = 1.03$$

At a discharge of 1500 cfs,

$$m = \frac{(500 \times 1.5) + ((870 - 500) \times 0.4) + ((1500-870) \times 1.3)}{1500}$$

$m = 1.15$

At a non-tabulated discharge such as 1000 cfs, the value of m is interpolated between values of m at the higher and lower tabulated discharges.

DEFINITION AND SIGNIFICANCE OF x

The coefficient x in equation 2-1 is an important parameter in the Modified Att-Kin routing procedure.

$$Q = x A^{m}$$

Holding m constant, the average flow velocity is

$$V = Q / A = x A^{m} / A = x (A^{m-1})$$

The coefficient x is a proportionality constant relating area and velocity. If x is small, velocity is small and if x is large, velocity is large.

If x is small, there must be more flow area at the cross section to maintain a discharge than if x were larger. Increased flow area translates to increased storage because storage equals length times flow area.

On the other hand, holding m constant, the smaller value of x will result in more attenuation of the peak discharge. This is due to reduced velocity and increased storage in the reach.

GUIDELINES FOR ESTIMATING x AND m

It is optional in TR-20 to enter x and m value for the equation $Q = xA^m$ to represent the Q and A relationship for a reach. These input values may be derived in several ways. One method would involve plotting Q and A points on log-log paper and visually fitting a line to the data. The slope of the line is m and the intercept at A = 1 is x. A second method would involve fitting Q and A data with a power curve function by some statistical method (method of averages or least squares, for example). A third method involves use of guidelines for estimating x and m given in this section.

This section includes equations and nomographs for estimating x and m values for trapezoidal and triangular cross sections. These guidelines based on simplified cross sections require a minimum of topographic and roughness data. They are for use in absence of detailed cross section information for a reach where a routing is desired. If detailed information is available, a rating table should be calculated and the resulting table entered as a cross section table. A limitation of these guidelines is that they are based on one segment cross sections (one roughness value). A channel or flood plain cross section may be approximated by a triangular or trapezoid shape. For example, if peak discharges are expected to be contained in a channel, a triangular or trapezoidal representation could apply. If the channel carries insignificant flow with respect to the flood plain, the flood plain may be given a simple geometry.

Both x and m values from these guidelines will not accurately represent x and m for complex multi-segment cross sections with varying roughness. These types of cross sections generally exhibit changes in slope of the discharge-area curve (plotted on log-log paper).

The assumption of normal flow with constant slope for all discharges was used in developing these guidelines. This is a reasonable assumption for small streams with moderate to steep slopes, but is not reasonable for flat slopes with backwater influences.

The equations for x and m for a trapezoidal cross section are based on the assumption that discharge-area data plot as a straight line on log-log paper. The purpose of this section is to calculate x and m such that one power curve will fit the actual flow and area data for all discharges at the cross section.

Substituting A = 1 into Manning's equation (2-5) results in

$$Q = x = 1.49\ s^{1/2} / n / p^{2/3} \qquad \text{Eq. 2-7}$$

When the bottom width (B) of the channel is greater than or equal to five feet the approximation p = B may be made. This results in the equation

$$x = 1.49\ s^{1/2} / n / B^{2/3} \qquad \text{Eq. 2-8}$$

where:
- s = slope ft/ft
- n = Mannning's roughness coefficient
- B = Bottom width (ft.)

Figure 2-1 is a nomograph to solve equation 2-8 for x with s, n and B given.

In equation form, the slope equals

$$m = \frac{\log Q(2) - \log Q(1)}{\log A(2) - \log A(1)} = \frac{\log [\ Q(2) / Q(1)\ ]}{\log [\ A(2) / A(1)} \qquad \text{Eq. 2-9}$$

Substituting Q(1) = x and A(1) = 1

$$m = \log(\ Q(2) / x) / \log A(2) \qquad \text{Eq. 2-10}$$

Substituting $Q(2) = \frac{1.49}{n}\ A(2)\ (\ A(2) / p\ )^{2/3}\ s^{1/2}$

and $\dfrac{1.49\ s^{1/2}}{n\ B^{2/3}}$

the equation for m becomes

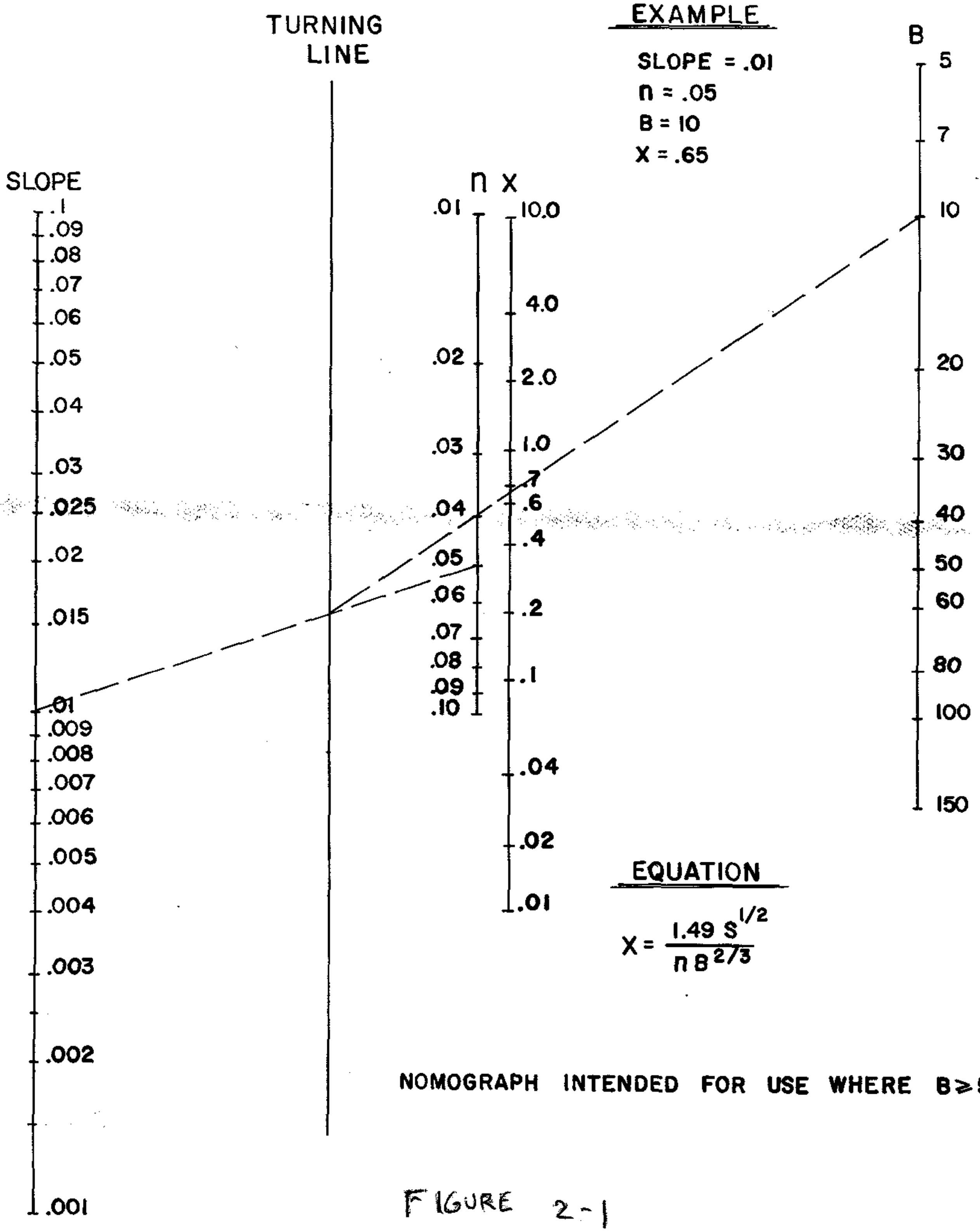
$$m = \frac{\log [\ A(2)^{5/3}\ (\ 1 / (\ 1 + 2\ D / B \sqrt{1 + Z^2}\ )^{2/3}\ )]}{\log A(2)} \qquad \text{Eq. 2-11}$$

where:
- D = maximum depth of channel, ft
- B = bottom width, ft
- Z = side slope, ft/ft
- $A(2) = BD + ZD^2$

NOMOGRAPH FOR ESTIMATING X FOR A TRAPEZOIDAL CHANNEL

$Q = XA^{M}$

SLOPE = BOTTOM SLOPE (FT/FT)
n = MANNING'S n
B = BOTTOM WIDTH (FT)



EXAMPLE

SLOPE = .01
n = .05
B = 10
X = .65

EQUATION

$$X = \frac{1.49\, S^{1/2}}{n\, B^{2/3}}$$

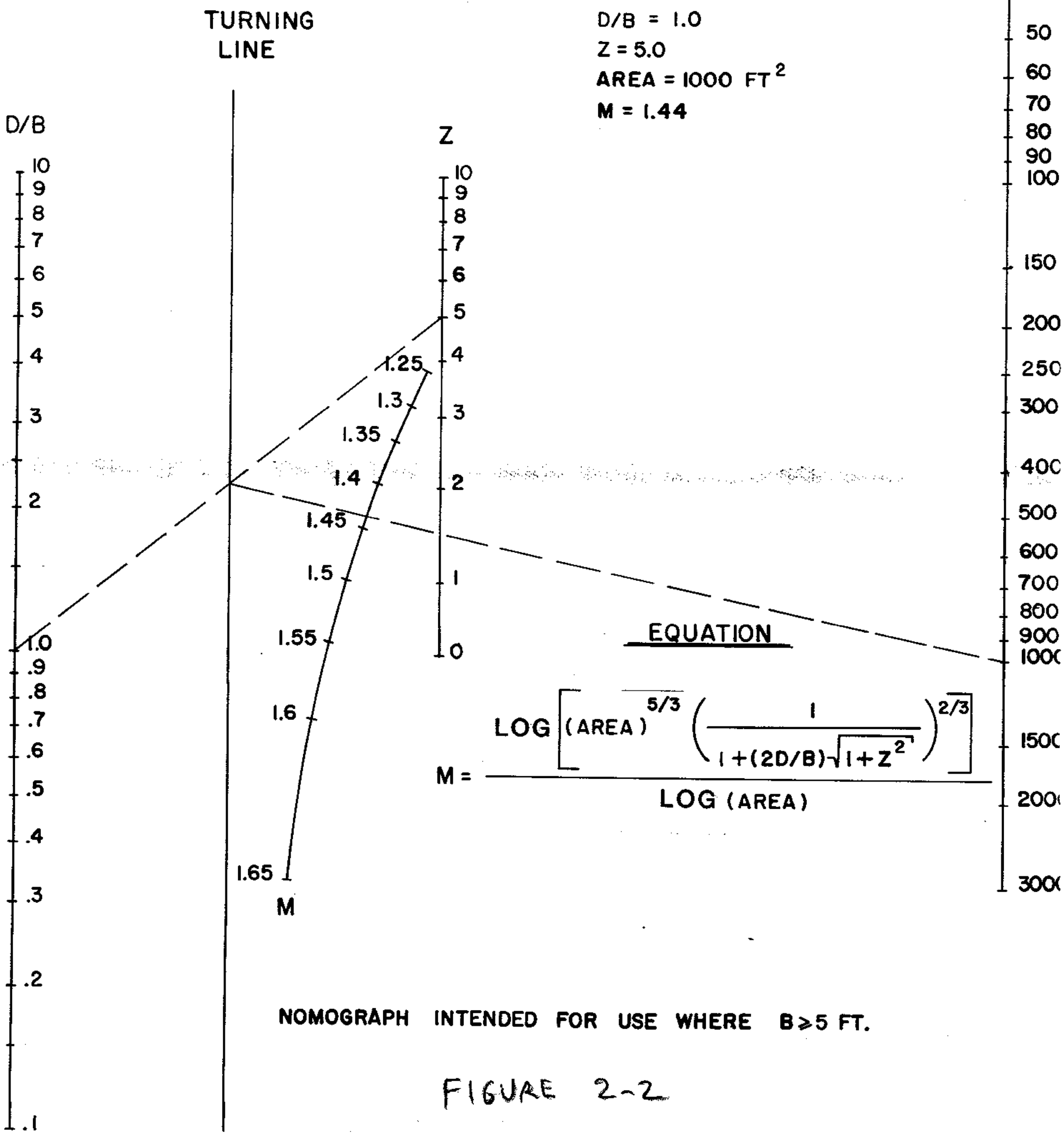
NOMOGRAPH INTENDED FOR USE WHERE B ≥ 5 FT.

FIGURE 2-1

NOMOGRAPH FOR ESTIMATING M FOR A TRAPEZOIDAL CHANNEL

$$Q = XA^M$$

D = MAXIMUM DEPTH (FT)
B = BOTTOM WIDTH (FT)
Z = SIDE SLOPE
AREA = BD + ZD^2



EXAMPLE

D/B = 1.0
Z = 5.0
AREA = 1000 FT^2
M = 1.44

EQUATION

$$M = \frac{\mathrm{LOG}\left[(\mathrm{AREA})^{5/3}\left(\frac{1}{1+(2D/B)\sqrt{1+Z^2}}\right)^{2/3}\right]}{\mathrm{LOG}\ (\mathrm{AREA})}$$

NOMOGRAPH INTENDED FOR USE WHERE B ≥ 5 FT.

FIGURE 2-2

Figure 2-2 is a nomograph for solution of equation 2-11.

The values of x and m for rectangular cross sections may estimated by using side slope ratio (Z) of zero.

Example 1

Estimate x and m for the channel with the following characteristics.

bottom width = 10 feet
side slopes = 5:1
maximum depth = 8 feet
n = .04
bottom slope = 0.5%

Using equation 2-8 to estimate x,

$$x = \frac{1.49 \quad (0.005)^{1/2}}{.04 \quad (10)^{2/3}} = 0.57$$

$$AREA = BD + ZD = (10 \times 8) + (5 \times 8) = 400$$

Using equation 2-11 to estimate m,

$$m = \frac{\log \left[400^{5/3} \left(1 / \left(1 + \left(2 \times 8 / 10 \sqrt{1 + 5^2} \right) \right) \right) \right]}{\log 400} = 1.42$$

The discharge-area equation is

$$Q = 0.57 A^{1.42}$$

The x and m can be entered into TR-20 if the trapezoidal cross section is representative of the reach.

For a triangular cross section, Manning's equation may be put in the form $Q = xA^m$ through the following derivation.

$$A = Z D^2$$

$$p = 2D\sqrt{1 + Z^2}$$

where: Z = side slope ratio, ft/ft
D = depth, ft
A = area, ft
p = wetted perimeter ft.
Z and D are defined as shown on Figure 2-3.

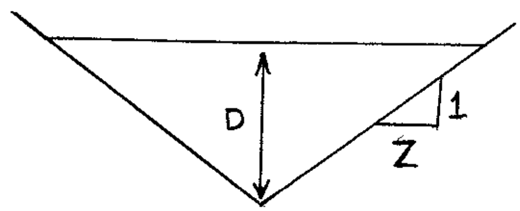


Figure 2-3 Triangular cross section

Substituting into Manning's equation and simplifying,

$$Q = \frac{0.94\ s^{1/2}\ Z^{1/3}\ A^{4/3}}{n\ (Z^2 + 1)^{1/3}} \qquad \text{Eq. 2-12}$$

if Z > 3 the equation 2-12 may be approximated by

$$Q = \frac{0.94\ s^{1/2}\ A^{4/3}}{n\ Z^{1/3}} \qquad \text{Eq. 2-13}$$

Therefore, for a triangular cross section

$$x = \frac{0.94\ s^{1/2}}{n\ Z^{1/3}} \qquad \text{Eq. 2-14}$$

For all triangular sections, assuming uniform roughness and normal flow,

$$m = 4/3 \qquad \text{Eq. 2-15}$$

An example of application of the triangular cross section guidelines follows.

Example 2

From a topographic map, the following data is obtained. Slope of channel bottoms is

$$s = \frac{10 \text{ ft}}{10000 \text{ ft}} = 0.001$$

At one contour interval (10 feet) above the channel bottom the top width of the cross section is measured to be 800 feet. Therefore,

$$Z = 1/2 \quad 800 \text{ ft} / 10 \text{ ft} = 40$$

The n value (based on vegetation and obstructions in the cross section) is estimated to be 0.05.

Therefore, using equation 2-14 to estimate x,

$$x = \frac{0.94 \quad (0.001)^{1/2}}{0.05 \quad (40)^{1/3}} = 0.17$$

From equation 2-15, m = 1.33

The equation for the discharge-area relationship is

$$Q = 0.17 \; A^{1.33}$$

The x and m can be entered into TR-20 if the triangular cross section is representative of the reach.

REACH LENGTH GUIDELINES

Reach Length in Relation to Attenuation

Reach routings using the Modified Att-Kin model approach a kinematic model routing if the reach lengths are short. The inflow hydrograph is essentially translated in time with little or no attenuation of the peak.

For a given valley cross section and inflow hydrograph, the attenuation increases as the reach length increases. To illustrate this concept, consider the length factor k* (as defined in Chapter 1).

$$k^* = QI^m / k / VI^m$$

where: QI = peak of runoff hydrograph, cfs.
VI = volume of runoff hydrograph, cubic ft.
k,m = coefficients of valley storage discharge relationship, $Q = kS^m$.

Using the valley storage-discharge equation and the flow area-discharge equation, it can be shown that

$$k = x / L^m \qquad \text{Eq. 2-16}$$

Substituting the value of k from equation 2-16 into 2-15,

$$k^* = \frac{QI \; L^m}{x \; VI^m} = \frac{QI}{x \; VI^m} \; L^m \qquad \text{Eq. 2-17}$$

Thus, for a given inflow hydrograph (with QI and VI) and valley cross section (with x and m), the value of k* increases with increasing reach length.

Figure 2-4 shows a relationship of k* versus Q*. Q* is a measure of peak flow reduction or attenuation.

$$Q^* = (Qo - Qb) / (QI - Qb)$$

where: Qo = peak of outflow hydrograph, cfs.
QI = peak of inflow hydrograph, cfs.
Qb = baseflow, cfs.

Figure 2-4 shows typical k* versus Q* curves for two given inflow hydrographs and valley cross sections as well as the limits on k* and Q* defined by kinematic routing, conservation of mass and routing through a linear reservoir. The linear reservoir limit is unique for a particular hydrograph and valley configuration. As the routing reach length increase, k* increases and Q* decreases. k* versus Q* plots for other hydrographs and valley cross sections could plot anywhere below the kinematic and conservation of mass limits plotted on Figure 2-4 and above a unique linear reservoir routing limit.

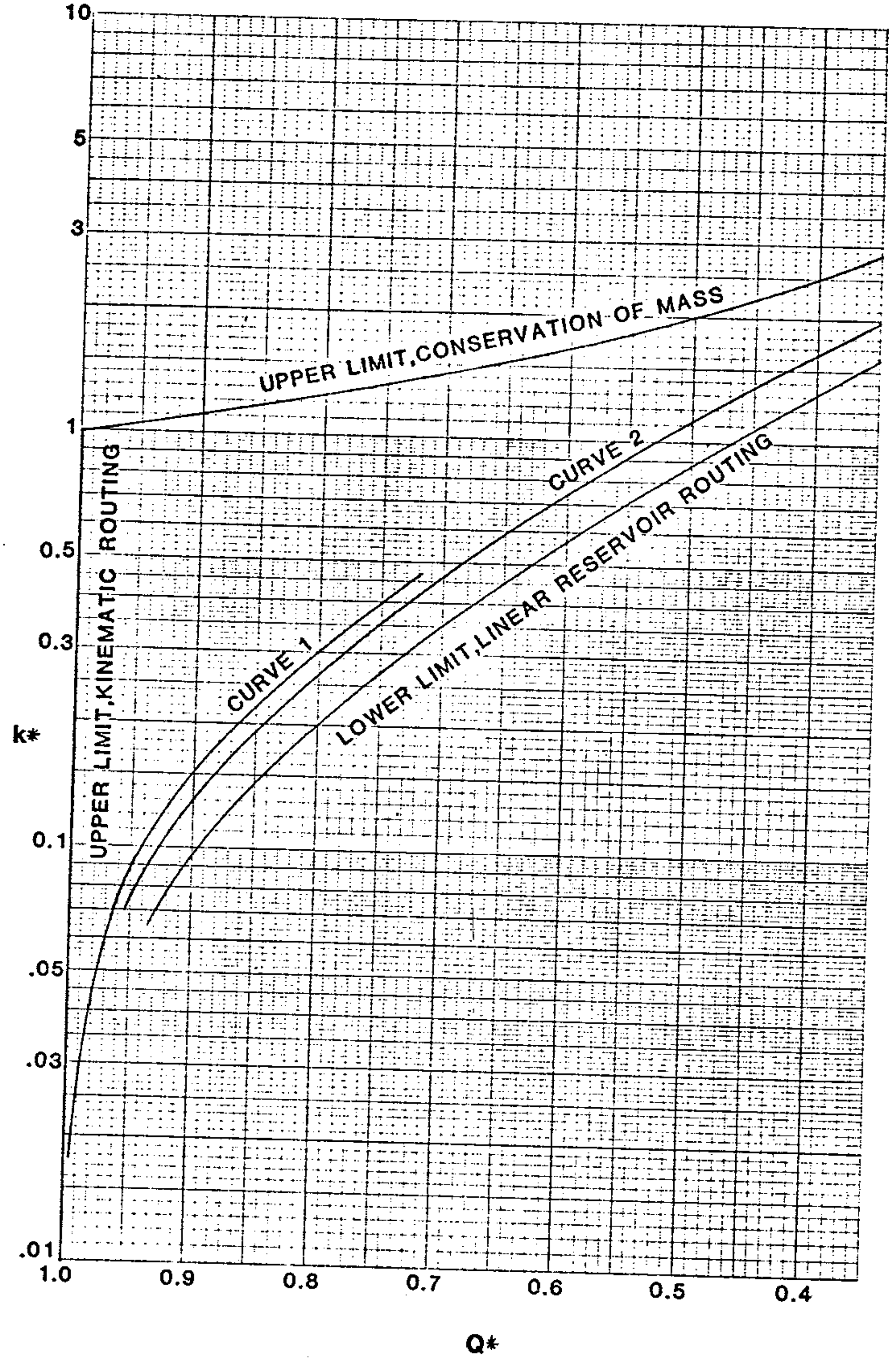


FIGURE 2-4

ATTENUATION VERSUS LENGTH FACTOR

Data used to calculate curve 1 include:
QI = 4,000 cfs, VI = 1,221 ac-ft
x = 0.4 and m = 1.3 for lengths ranging from 2,000 feet to 25,000 feet.

Data used to calculate curve 2 include:
QI = 4,000 cfs, VI = 888 ac-ft
x = 0.15 and m = 1.3 for lengths ranging from 2,000 feet to 25,000 feet.

From these two examples, it can be seen that with different values of VI and x that a significant difference in peak outflow will result.

For curve 1, a reach length of 2,000 feet would result in k* = .017 (from Eq. 2-17) and less than 1% reduction in peak discharge.

With this small attenuation, the routing approaches a kinematic routing. However, with curve 2, a reach length of 2,000 feet will result in k* = .07 (from Eq. 2-17) and a peak flow reduction of almost 5%. This routing has much less kinematic effect than that for curve 1.

The reach length at which a Modified Att-Kin routing approaches a kinematic routing depends on the four variables QI, VI, x, and m.

The value of k* (which is related to each of the four variables) determines to what degree the Modified Att-Kin routing attenuates an inflow hydrograph.

Reach Length in relation to Travel Time

The following discussion will explain the general reach length guidelines and provide details on selecting a main time increment and/or reach lengths for routings.

As a general guideline, if a reach is selected such that the kinematic travel time is longer than the main time increment, a kinematic routing (one with no attenuation and only translation) will be avoided.

The relationship of main time increment, peak travel time (kinematic), reach length and kinematic routing effect can be illustrated through equation 1-17.

$$Cr = 2\Delta t \;/\; (2K + \Delta t) \qquad \text{Eq. 1-17}$$

where: Cr = Modified Att-Kin routing coefficient.

Δt = main time increment, hours.
K = slope of storage discharge curve, hours.

From the routing equation relating inflow to outflow discharges, if Cr = 1.0, the outflow discharges and inflow discharges are equal. Thus, pure translation of the inflow hydrograph occurs when Cr equals 1.0.

K is also the kinematic travel time of the peak discharge through the reach for short and moderate reach lengths (k* less than 1.0).

$$K = L / (m V) \qquad \text{Eq. 2-18}$$

where: L = reach length, ft.
m = exponent of discharge storage relationship
V = average velocity of peak inflow, ft/sec

Lower and upper limits of m in TR-20 are 1.0 and 2.0.

In order to avoid a kinematic translation of the inflow hydrograph,

$$K > \Delta t / 2 \qquad \text{Eq. 2-19}$$

The travel time should be greater than one half the main time increment. In terms of reach length and travel time,

$$L_{min} / (m V) = \Delta t / 2 \qquad \text{Eq. 2-20}$$

where: Lmin = minimum acceptable reach length, feet.

Solving equation 2-20 for Lmin,

$$L_{min} = m V \Delta t / 2 \qquad \text{Eq. 2-21}$$

In Equations 2-20 and 2-21, if V is in feet per second then Δt must be in seconds.

The reach length for which the travel time equals one-half the main time increment is the minimum reach length to select in order to avoid kinematic translation. At this length, the peak travel time would be rounded up to the main time increment. If a number of these reaches were routed consecutively, the time error would accumulate. It should be realized that in certain circumstances such as routing structure outflows, routing hydrographs on steep streams with high velocities, or routing 10 day duration hydrographs that this length may be impractical to use.

In order to minimize error in rounding the peak travel time to the main time increment (if the kinematic travel time is less than the main time increment), the reach length should be selected such that the travel time is greater than or equal to the main time increment.

$$K = Lr / (m V) = \Delta t \qquad \text{Eq. 2-22}$$

where: Lr = minimum recommended reach length, ft.

$$Lr = \Delta t \; m V \qquad \text{Eq. 2-23}$$

If $K = \Delta t$, then

$$Cr = 2 \Delta t / (2 \Delta t + \Delta t) = 2/3 \qquad \text{Eq. 2-24}$$

If $K > \Delta t$ then $Cr < 2/3$ and the reach length is in the recommended range. If Cr is between 2/3 and 1.0 the length is in the acceptable range. If Cr is 1.0 the reach length is too short or Δt is too large.

Figure 2-5 is a nomograph for selecting the main time increment, on the minimum reach length (Lmin) or the minimum recommended reach length (Lr).

<u>Example 3</u>

At a particular cross section mV is estimated to be 4 ft/sec based on normal flow assumptions. If the desired main time increment is .20 hours, from Eq. 2-20

Lmin = 1/2 (4 ft/sec) x (0.2 hr x 3600 sec/hr) = 1440 ft.

From Equation 2-23,

Lr = 4 ft/sec x .20 hr x 3600 sec/hr = 2880 ft.

A reach length shorter than 1440 feet is considered too short, a reach length between 1440 and 2880 feet is acceptable and a reach length greater than 2880 feet is recommended.

If Lmin is too long for a particular application of TR-20 then the main time increment may be reduced to give a shorter Lmin.

NOMOGRAPH RELATING MAIN TIME INCREMENT WITH REACH LENGTH

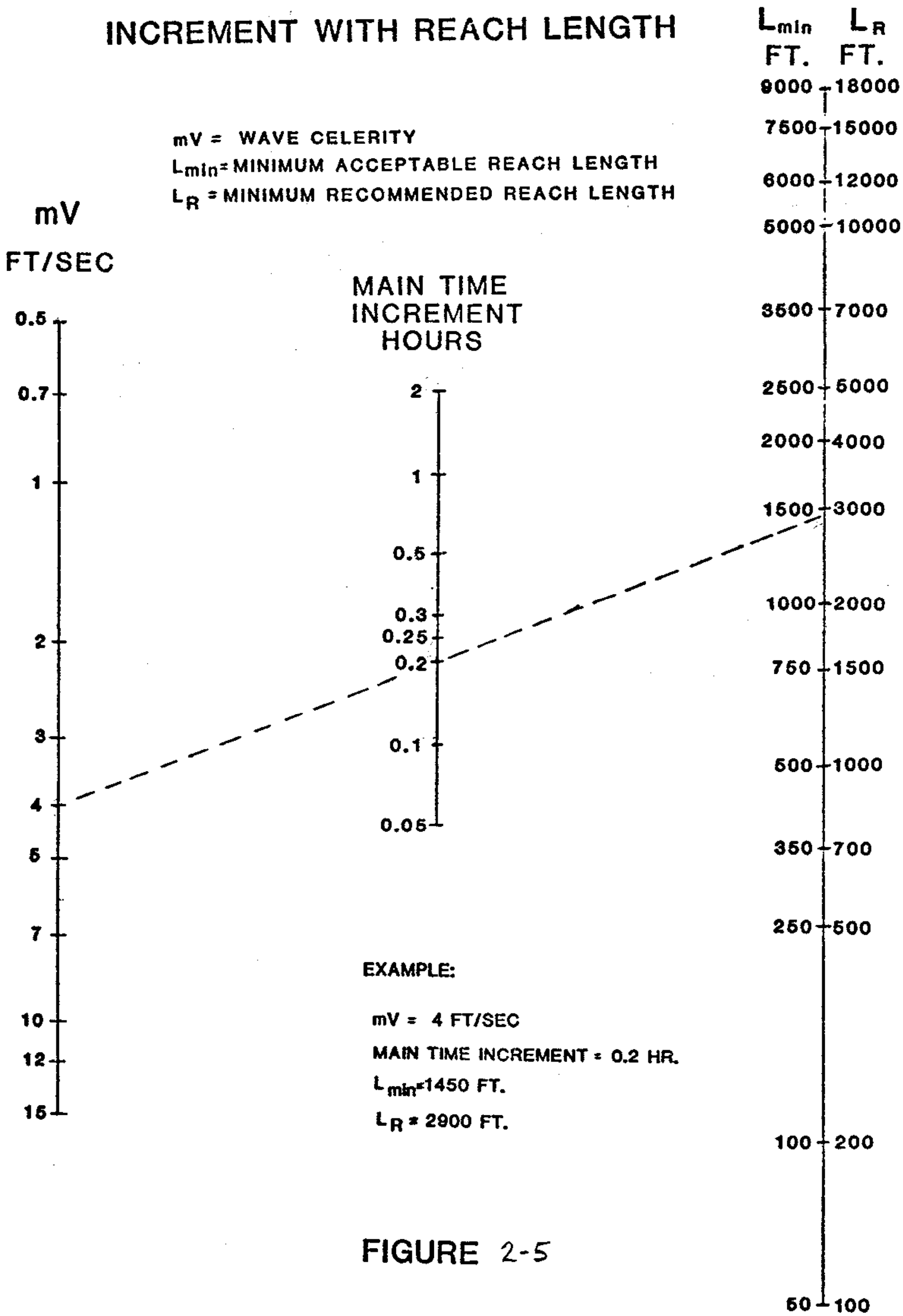


FIGURE 2-5

Example 4

If reach lengths are already determined, the guidelines can be used to select an appropriate main time increment.

Water surface profiles have been run for the streams in a watershed. The values of mV are estimated at selected cross sections in a watershed and the maximum is 3 ft/sec. The reach length used with this cross section is 3000 feet. What main time increment should be selected ?

$$K = L / (m V) = 3000 / 3 = 1000 \text{ sec or } 0.28 \text{ hour.}$$

Since it is recommended that K be greater than the main time increment, a time increment of 0.25 hour or less is recommended. A main time interval over 0.5 hour should not be selected. It should be noted that other factors in TR-20 impact the selection of the main time increment and should be considered also. One of these factors is time of concentration of sub-watersheds. Another is possible truncation of hydrographs due to long hydrograph duration and the limited number of coordinates saved in a hydrograph.

In the TR-20 User Manual it is recommended not to change the main time increment within a pass (analysis of one storm). However, if there is need to maintain short reaches, the main time increment may be changed within a pass with consideration given to the possibility of truncating the hydrograph. The main time increment may also be increased within a pass to avoid truncation of hydrographs at downstream reaches which have long travel times and extend the duration of the hydrograph.

References

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Huang, Y. H. 1978. Channel Routing by Finite Difference Method, Journal of the Hydraulics Division, ASCE, Vol. 104, Oct. 1978.

Theurer, F. D. and Comer, G. H., Verification of SCS Dam-Breach Routing Procedure, Presented at 1979 Winter Meeting of American Society of Agricultural Engineers.

CHAPTER 3

EXAMPLES OF ROUTINGS USING THE MODIFIED ATT-KIN METHOD

Introduction

Several examples are included to show computations of flood routings using the Modified Att-Kin method. These assume that the cross section rating table, reach length, and inflow hydrograph are already available for use. For assistance in developing these, refer to SCS Engineering Handbook Chapter 31 (Water Surface Profiles) and NEH-4 Chapters 6 (Stream Reaches and Hydrologic Units) and 16 (Hydrographs).

Two examples are included. Example 1 illustrates a routing with coefficients x and m given. Example 2 illustrates a routing with a complex cross section with rating table given.

Example 1

The rating table for the reach (length of 2600 feet) can be represented by the values of x = 0.3 and m = 1.3. The discharge-area relation is:

$$Q = 0.3 \ A^{1.3}$$

The inflow hydrograph is:

Time	Discharge,cfs	Time	Discharge,cfs
0.0	0.0	6.75	310
.25	50	7.0	290
.5	110	7.25	280
.75	180	7.5	270
1.0	240	7.75	260
1.25	300	8.0	250
1.5	400	8.25	240
1.75	500	8.5	230
2.0	600	8.75	220
2.25	730	9.0	210
2.5	750	9.25	200
2.75	730	9.5	190
3.0	700	9.75	180
3.25	670	10.0	170
3.5	650	10.25	160
3.75	630	10.5	140
4.0	610	10.75	120
4.25	580	11.0	100
4.5	540	11.25	80

Time	Discharge,cfs	Time	Discharge,cfs
4.75	500	11.5	60
5.0	450	11.75	50
5.25	430	12.0	40
5.5	410	12.25	30
5.75	390	12.5	20
6.0	370	12.75	10
6.25	350	13.0	0
6.5	330		

Solution: Compute k*.

$$k = x / L^{m} = 0.3 / (2600^{1.3}) = 1.09 \times 10^{-5}$$

VI = volume of inflow in cubic feet

VI = sum of hydrograph coordinates x 0.25 x 3600

$$VI = 16310 \times 0.25 \times 3600 = 1.47 \times 10^{7} \text{ cubic feet.}$$

$$k^* = QI / k / VI^{m} = 750 / 1.09 \times 10^{-5} / ((1.47 \times 10^{7})^{1.3})$$

$$k^* = 0.033$$

Since k* is less than 1.0, use the peak inflow as the reference discharge. Compute velocity for this discharge.

$$750 = x A^{m} = 0.3 A^{1.3} \qquad A = 411 \text{ sq. ft.}$$

$$V = 750 / 411 = 1.825 \text{ ft/sec}$$

Compute travel time K.

$$K = L / (mV) = 2600 / (1.3 \times 1.825) = 1096 \text{ sec} = .3044 \text{ hr}$$

Compute routing coefficient, Cr.

$$Cr = 2 \Delta t / (2 K + \Delta t) = 0.5 / (0.6088 + .25) = .582$$

Do the routing using the equation:

$$O2 = Cr\ I1 + (1 - Cr)\ O1$$

The following table shows results of the routing.

Time	Inflow,cfs	Outflow,cfs	Remarks
0	0	0	
0.25	50	0	
0.5	110	29	29 = 0.582 x 50
0.75	180	76	76 = 0.582 x 110 + 0.418 x 29
1.0	240	137	137 = 0.582 x 180 + 0.418 x 76
1.25	300	197	197 = 0.582 x 240 + 0.418 x 137
1.5	400	257	
1.75	500	340	
2.0	600	433	
2.25	730	530	
2.5	750	647	Peak Inflow
2.75	730	707	
3.0	700	720	Peak Outflow
3.25	670	708	
3.5	650	686	
3.75	630	665	
4.0	610	645	
4.25	580	624	
4.5	540	599	
4.75	500	564	
5.0	450	527	
5.25	430	482	
5.5	410	452	
5.75	390	427	
6.0	370	406	
6.25	350	385	
6.5	330	365	
6.75	310	344	
7.0	290	324	
7.25	280	304	
7.5	270	290	
7.75	260	278	
8.0	250	268	
8.25	240	257	
8.5	230	247	
8.75	220	237	237 = 0.582 x 230 + 0.418 x 247
9.0	210	227	
9.25	200	217	
9.5	190	207	
9.75	180	197	
10.0	170	187	
10.25	160	177	
10.5	140	167	
10.75	120	151	
11.0	100	133	
11.25	80	114	
11.5	60	94	
11.75	50	74	
12.0	40	60	
12.25	30	48	
12.5	20	38	
12.75	10	27	

Time	Inflow,cfs	Outflow,cfs
13.0	0	17
13.25	0	7
13.5	0	3
13.75	0	1
14.0	0	0

Compare Δtps and Δtp to determine if hydrograph needs to be shifted in time. Δtps is the time difference between the inflow and outflow peaks.

$$\Delta tps = 3 - 2.5 = 0.5 \text{ hour}$$

Use equation 1-25 to compute Δtp.

At a discharge of 720 cfs, the flow area (A) is:

$$A = (720 / 0.3)^{1/1.3} = 398.3 \text{ sq.ft.}$$

$$Spo = L \times A = 2600 \times 398.3 = 1.036 \times 10^{6} \text{ cubic ft.}$$

$$\Delta tp = \frac{1.036 \times 10^{6} \left((750 / 720)^{1/1.3} - 1 \right)}{720 \left((750 / 720) - 1 \right)} = 1100 \text{ sec}$$

$$\Delta tp = 0.30 \text{ hours.}$$

Since Δtp is less than Δtps, no adjustment in time is needed and the routing is completed.

Example 2

This example uses a rating table to represent the discharge and flow area of the reach. The reach length is 5400 feet and the rating table is as follows.

Q cfs	A sq.ft.
0	0
50	28.3
300	94.3
500	184.4
750	333.0
1070	483.8
1500	655.2

The inflow hydrograph has a time increment of 0.1 hour. The discharges are:

0	16	31	47	63
80	98	116	134	165
220	308	429	581	761
951	1119	1243	1318	1347
1338	1294	1223	1134	1030
922	824	742	672	611
557	509	466	428	394
365	338	314	293	275
257	239	221	203	185
167	149	131	114	97
80	65	50	36	22
10	0			

Solution: Compute m at each discharge in the rating table. The following table shows the results.

Q	A	Log Slope	Product of slope and difference in Q	m
0	0			1.49
		1.49 *	74.5	
50	28.3			1.49
		1.49	372	
300	94.3			1.49
		0.76	152	
500	184.4			1.20
		0.686	172	
750	333.0			1.03
		0.95	304	
1070	483.8			1.00
		1.114	479	
1500	655.2			1.04

* Note: since this slope is not defined, the slope from the first two non-zero discharges is used.

Compute k*. First, the rating coefficient x and exponent m need to be computed.

At a discharge of 1347 cfs (peak inflow) interpolate flow area and m from the original rating table. This interpolation is log-log.

$$A = \exp \left[\log 483.8 + \frac{\log (1347/1070)}{\log (1500/1070)} \log (655.2/483.8) \right]$$

$A = 595.2$ sq. ft.

$$m = \exp \left[\log 1.0 + \frac{\log (1347/1070)}{\log (1500/1070)} \log (1.04/1.0) \right]$$

$m = 1.027$

Determine x from the equation $Q = x\,A^{m}$.

$1347 = x \; 595.2^{1.027}$ $\qquad\qquad x = 1.905$

Determine VI (volume of inflow).

VI = sum of hydrograph coordinates * 0.1 * 3600

$VI = 24782 * 0.1 * 3600 = 8.92 \times 10^{6}$ cubic ft.

$$k^* = 1347 \,/\, [\, 1.905 \,/\, (5400^{1.027}) \, (8.92 \times 10^{6})^{1.027} \,]$$

$k^* = 0.35$

Since k* is less than 1.0, use the peak inflow with associated flow area and m to compute K.

$K = L / (m\,Q / A) = 5400 / (1.027 * 1347 / 595.2)$

$K = 2323$ sec $= .645$ hour

Next, compute routing coefficient, Cr.

$Cr = 0.2 / (2 * .645 + 0.1) = 0.144$

Do the routing using the equation $O2 = Cr\, I1 + (1-Cr)\, O1$.

Time	Inflow	Outflow	Time	Inflow	Outflow
0	0	0	1.9	1347	711.4
0.1	16	0	2.0	1338	802.6
0.2	31	2.3	2.1	1294	879.7
0.3	47	6.4	2.2	1223	939.3
0.4	63	12.2	2.3	1134	980
0.5	80	19.5	2.4	1030	1002.2
0.6	98	28.2	2.5	922	1006
0.7	116	38.2	2.6	824	994.1
0.8	134	49.4	2.7	742	969.6
0.9	165	61.6	2.8	.	.
1.0	220	76.5	2.9	.	.
1.1	308	97.2	3.0	.	.
1.2	429	127.6	3.1	.	.
1.3	581	171	3.2	.	.
1.4	761	230			
1.5	951	306.5			
1.6	1119	399.3			
1.7	1243	502.9			
1.8	1318	609.4			

The peak outflow is 1006 cfs and occurs at 2.5 hours. Using the equation $Q = x A^{m}$ compute flow area for 1006 cfs.

$$1006 = 1.905 * A^{1.027} \qquad A = 448 \text{ sq. ft.}$$

Interpolate flow area for 1006 cfs from original rating table.

$$A = \exp \left[\log 333 + \frac{\log (1006/750)}{\log (1070/750)} \log (483.8/333) \right]$$

$$A = 454 \text{ sq. ft.}$$

Interpolate m at a discharge of 1006 cfs.

$$m = \exp \left[\log 1.03 + \frac{\log (1006/750)}{\log (1070/750)} \log (1.0/1.03) \right]$$

$$m = 1.01$$

The flow area of 448 is within five percent of 454 and m of 1.027 is within 0.05 of 1.01 so the routing is satisfactory to this point. If the flow area or m were outside these tolerances, the flow area and m would be adjusted and the routing equation solved again.

The timing of the outflow hydrograph needs to be checked to determine if a time shift is needed.

The time difference between the peak inflow and outflow is 2.5 - 1.9 = 0.6 hours. This is Δtps.

The kinematic travel time is :

$$Spo = L\ A = 5400 \text{ feet} \times 448 \text{ sq feet} = 2.42 \times 10^{6} \text{ cubic ft}$$

$$\Delta tp = \frac{2.42 \times 10^{6}}{1006} \cdot \frac{\left(\left(\frac{1347}{1006}\right)^{1/1.027} - 1\right)}{\left(\frac{1347}{1006} - 1\right)}$$

Δtp = 2333 sec = 0.65 hour.

Since Δtp is greater than Δtps, an adjustment of the outflow hydrograph is needed. The difference of 0.05 hour is less than the main time increment of 0.1 hour, so the outflow hydrograph is moved one time increment. All hydrograph points will have 0.1 hour added to their time of occurrence. The outflow peak will occur at 2.6 hours instead of 2.5 hours.